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STABILITY RESEARCH ON PARACHUTES USING DIGITAL AND ANALOG COMPUTERS

R. Ludwig

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The computation of numerous examples of a special type (numerically about 80 cases were considered) shows that the oscillations of a parachute show a certain typical type of behavior which is characteristic of nonlinear oscillations. A qualitative agreement with experiments was achieved in a number of respects. Quantitative comparative investigations still could not be carried out because until now it still was not possible to carry out drop experiments with chutes of the considered type.

In addition to the information which the experimenter obtains on different properties of parachute oscillation, it appears to be particularly important that this is a case where for the investigation of the dynamic behavior a nonlinear computation is the only approach which can give an unobjectionable description of the process. The available experimental data, such as wind tunnel measurements for asymmetrical chutes in 6 components, measurements of the entrained air mass, etc., should be used in further broadening of theoretical investigations.

1. Introduction /1*

In the consideration of dynamic problems in flight mechanics, it was customary at an earlier time to linearize the problem, that is, the effect of small perturbations was considered. The system of linear differential equations following from this approach also had the pleasant property that with a relatively minor number of computations it was possible to draw conclusions concerning frequencies and attenuations. The admissibility of linearization in many cases was questionable from the beginning. Now, on the other hand, in most cases there has been a changeover to nonlinear computations. It is accepted, thus, that the volume of computations will be very greatly increased and that it scarcely is possible to draw any general conclusions; conclusions can be drawn only from numerous examples in which certain parameters are varied. Only by use of the modern tools of analog and digital computers has it become possible to compute the dynamic problems of flight mechanics in this universality.

In the investigation of the dynamic stability of parachutes, which will be

*/Numbers in the margin indicate pagination in the original foreign text.

discussed here, until now work always has begun with linearized equations of motion. Even though the first publication known to the author had already appeared in 1918 [1], a communication published by W. G. S. Lester (1962) [3] made no mention of it. As will be assumed here in advance, the result of linearization in this case is particularly complicated. The differential equation for the perturbation of velocity is split off from the other differential equations and gives a monotonic attenuation of a perturbation, but not a periodic attenuation with the frequency of the oscillating chute. Here, this means that linearization, regardless of whether the application of the theory of small oscillations is admissible, leads to a deficient description of the physical behavior.

From the point of view of parachute technology, the attainment of good dynamic stability is of great importance. Regardless of the objective of the chute -- whether for saving a pilot, for ejection, for braking the landing of an aircraft or as a chute for stabilizing any kind of flight vehicle -- in all cases an insufficient stability will at least lead to difficulties or even de- /2 stroy the real purpose of the chute.

Moreover, the assumption of small perturbations also is scarcely realized in practice. The influence of a gust on a stably falling chute very easily can lead to deflections which no longer justify a linearization.

The aerodynamic values for resistance (drag), shear and moment in dependence on angle of attack, needed for computations, as known from wind tunnel measurements, likewise show a behavior which does not admit a linearized treatment, as is customary in flight mechanics, for example

$$C_M = C_{M_0} + \frac{\partial C_{M_0}}{\partial \alpha} \Delta\alpha$$

because in part $\partial C_M/\partial\alpha$ even varies in sign.

For the model calculations, which will be reported on here, the so-called personnel guide surface parachute will be used. For this parachute, whose prototype was developed during the Second World War by Prof. Heinrich at the Aeronautical Institute in Stuttgart, in Germany (Prof. Madelung) we have American wind tunnel measurements which were carried out at the Institute by Prof. Heinrich (University of Minnesota, USA).

In the numerous computed examples, we investigated the dependence of different parameters, such as the influence of the length of the shroud lines, the variation of the mass of the entrained air, and the density of the surrounding air (or altitude). Finally, the stable state of oscillation also was considered for the case in which, in a certain neighborhood of the angle of attack zero, the moment is not restoring (that is, $\partial C_M/\partial\alpha$ varies in sign in the corresponding region).

2. Notations /3

$\mathfrak{V}$, V, V_x, V_y	[m/sec]	= velocity vector, sum of velocity, components in a coordinate system related to the parachute
V_s	[m/sec]	= stable speed of descent
$\vec{\omega}$, ω	[sec^{-1}]	= angular velocity
$G_L = m_L g$	[kg]	= load on the parachute
m_L	[$\mathrm{kg{\cdot}sec^2/m}$]	= mass of the load
m_K	[$\mathrm{kg{\cdot}sec^2/m}$]	= air mass entrained by shroud
μ	[1]	= ratio of the entrained air mass to the mass of the load
ϑ	[1]	= angle of inclination of trajectory
γ	[1]	= longitudinal angle of inclination
α	[1]	= angle of attack
I_K, i_K	[$\mathrm{kg{\cdot}m{\cdot}sec^2}$, m]	= moment of inertia, radius of the shroud
s	[m]	= distance from midpoint of shroud to point of application of load
γ	[kg]	= internal force
$\mathfrak{P}$	[kg]	= external force
W	[kg]	= resistance
Q	[kg]	= shear
M	[kg·m]	= moment
C_W, C_Q, C_M	[1]	= resistance, shear and moment of shroud
$F = R^2\pi$	[$\mathrm{m^2}$]	= reference plane of parachute shroud for the air force
R	[m]	= radius of the parachute shroud /4

t	[sec]	= time
x, y	[m]	= coordinates of trajectory in a coordinate system related to the ground
g	[m/sec^2]	= acceleration of gravity
ρ	[kg·sec^2/m^4]	= air density

The time derivatives are denoted by a dot.

Subscripts: K = shroud; L = load.

3. Equations of Motion

The equations of motion will only be considered briefly [5]. The premises are:

a) The shroud-load system is rigid.

b) The motion occurs in a vertical plane passing through the axis of the chute.

c) The shroud entrains an air mass which is to be regarded as a sluggish but not as a heavy mass; it will be called the apparent (as in English) or entrained mass. On the other hand, the mass of the chute can be neglected.

d) The chute is acted upon by aerodynamic forces, resistance in the direction of the trajectory and the shear perpendicular to it, and an aerodynamic moment about an axis perpendicular to the plane of the trajectory. On the load, there should be only a negligibly small resistance, but no shear and no moment.

Now we will consider the force equations for the shroud and load separately (Figure 1):

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$$m_K(\dot{\mathfrak{v}}_K + \vec{\omega} \times \mathfrak{v}_K) = \mathfrak{T}_K + \mathfrak{P}_K \quad , \tag{1}$$

$$m_L(\dot{\mathfrak{v}} + \vec{\omega} \times \mathfrak{v}) = -\mathfrak{T}_L + \mathfrak{P}_L \quad , \tag{2}$$

and the equation of moment, related to L in a coordinate system related to the parachute

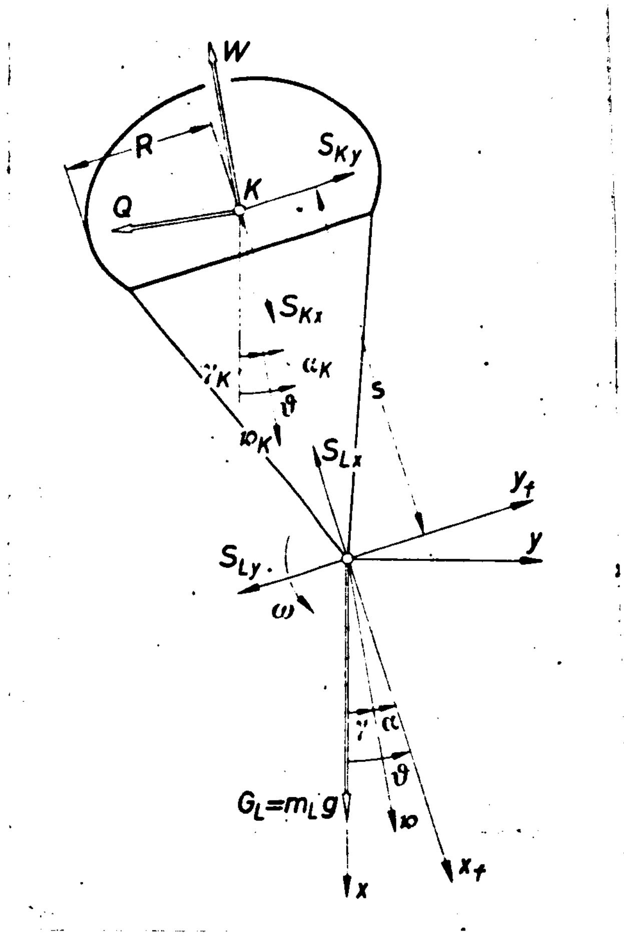

Figure 1. Notations.

$$I_K \dot{\omega} - m_K s \dot{v}_{Ky} - m_K s v_{Kx} \omega = -M_K \tag{3.1}$$

$$\dot{\vartheta} = \omega \tag{3.2}$$

It follows from the premise of rigidity of the shroud-load system that:

$$\gamma_K = \gamma_L \quad , \tag{4}$$

$$V_{Kx} = V_x \quad , \qquad (6) \quad V_{Ky} - V_y = -s\omega \quad , \tag{5 & 6}$$

from the figure we also have the geometrical notations

$$\vartheta = \gamma + \alpha = \gamma_K + \alpha_K \quad , \tag{7a}$$

$$tg\,\alpha = -V_y / V_x \quad , \qquad tg\,\alpha_K = -V_{Ky} / V_x \quad . \tag{7b}$$

As the operating external forces

$$P_{Kx} = -W \cos\alpha_K - Q \sin\alpha_K \; , \; P_{Ky} = W \sin\alpha_K - Q \cos\alpha_K \; , \tag{8}$$

$$P_{Lx} = m_L g \cos\vartheta \qquad , \; P_{Ly} = -m_L g \sin\vartheta \quad . \tag{9}$$

Then we put the aerodynamic forces and moments in the usual form: /6

$$\begin{aligned} W &= \frac{\varrho}{2} F V_K^2 c_W(\alpha_K) \quad , \\ Q &= \frac{\varrho}{2} F V_K^2 c_Q(\alpha_K) \quad , \\ M_K &= \frac{\varrho}{2} F\, 2R\, V_K^2 c_M(\alpha_K) \quad . \end{aligned} \tag{10}$$

If we substitute equations (4)-(10) into equations (1)-(3) and, in addition, introduce differential equations for the trajectory of the load and shroud, we obtain fifteen values for the complete description of the dynamic behavior of the chute, namely, for the motion of the load:

$$V, \; V_x, \; V_y, \; \omega, \; \vartheta, \; \gamma, \; \alpha, \; x, \; y \; ;$$

and for the motion of the shroud:

$$V_K,\ V_{Ky},\ \gamma_K,\ \alpha_K,\ x_K,\ y_K\ .$$

We now have a system of 6 differential equations and 9 algebraic notations.

$$(m_L + m_K)(\dot V_x - \omega V_y) + m_K s\omega^2$$
$$= m_L g\cos\vartheta - \frac{\rho}{2}F V_K^2\left[C_N(\alpha_K)\cos\alpha_K + C_Q(\alpha_K)\sin\alpha_K\right], \qquad (11)$$

$$(m_L + m_K)(\dot V_y + \omega V_x) - m_K s\dot\omega$$
$$= -m_L g\sin\vartheta + \frac{\rho}{2}F V_K^2\left[C_N(\alpha_K)\sin\alpha_K - C_Q(\alpha_K)\cos\alpha_K\right], \qquad (12)$$

$$m_K(i_K^2 + s^2)\dot\omega - m_K s(\dot V_y + \omega\dot V_x) = -\frac{\rho}{2}F\,2R V_K^2 C_M(\alpha_K), \qquad (13)$$

$$\dot\vartheta = \omega, \qquad (14)$$

$$\dot x = V\cos\gamma, \qquad (15)$$

$$\dot y = -V\sin\gamma, \qquad (16)$$

$$V_{Ky} = V_y - s\omega, \qquad (17)$$

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$$V_K = \sqrt{V_x^2 + V_{Ky}^2}, \qquad (18)$$

$$V = \sqrt{V_x^2 + V_y^2}, \qquad (19)$$

$$\alpha_k = \operatorname{arctg}(-V_{ky}/V_{kx}) \tag{20}$$

$$\alpha = \operatorname{arctg}(-V_y/V_x) , \tag{21}$$

$$\gamma_k = \vartheta - \alpha_k , \tag{22}$$

$$\gamma = \vartheta - \alpha , \tag{23}$$

$$x_k = x - s \cos\vartheta , \tag{24}$$

$$y_k = y + s \sin\vartheta \tag{25}$$

In addition, we have the initial conditions for time t = 0. We will assume that while the chute is in stable vertical motion with the speed

$$V_s = \sqrt{\frac{2G_L}{\rho F c_{w_0}}} \tag{26}$$

it is deflected laterally in the trajectory plane by a gust, or the like, by an angle ϑ_0 and then

$$t = 0 : \quad V = V_s , \quad \vartheta = \vartheta_0 , \quad \omega = \gamma = 0 ,$$
$$x_0 = y_0 = 0 , \quad x_{K0} = -s \cos\vartheta_0 , \quad y_{K0} = s \sin\vartheta_0 .$$

In the model computations, the equations (12) and (13) are transformed in such a way that there will be one equation each for $\dot{V}_y$ and $\dot{\omega}$

4. Computation Methods

The solutions of the system of 6 differential equations were obtained using both digital and analog computers.

With respect to the digital computation we note the following:

The computations first were carried out on an IBM 650 (AVA, Göttingen) and this year on a Siemens 2002 of the DFL. Therefore, the usual solution method of step integration by the Runge-Kutta method (fourth order) was used. The programming was accomplished using the symbolic SOAP or HASI programming languages. The aerodynamic values C_W, C_Q and C_M were taken from a table as a function of the angle of attack α_K and interpolated linearly. Since the interpolation of the three functions within a Runge-Kutta interval must be carried out four times, it is recommended that the search time be shortened using for this purpose a specially reserved index, an indicator of the last computed place so-to-speak, and from there on, above and below, seek the proportionately near-lying value. In certain computations extending over greater time intervals, the values are approximated by (fifth or sixth degree) polynomials (as direct or indirect functions), resulting in a further saving of time without a loss of accuracy. By trial and error, we determined suitable intervals Δt for attaining the required accuracy. In the case of longer trajectories, the value Δt = 0.05 sec was used, but only each 10th step was used. For increasing clarity, and also for shortening the computation time (at the time only the Siemens 2002 with punch tape printout (60 symbols/sec) was available), the computations were made, to be sure, with a floating point (10-digit mantissa), but also with a fixed point with a reasonable number of decimals set aside (for example, in the case of trajectory coordinates in meters -- 2 decimals). /8

Comments on Computations with the Analog Computer*

A PACE 231 R analog computer of Electronic Associates, Inc., was available.

For reduction of multiplication units, the aerodynamic values were used in a coordinate system related to the body and also were approximated in part by polynomials if the dependence on certain parameters was under investigation. Here we even went so far, for example, as to approximate the expression $C_x(\alpha_K) = C_x(-\tanh^{-1} V_y/V_x)$ through a polynomial $C_x{*}(V_y/V_x)$ in the pertinent region.

The results obtained were accurate to about 1%. The reason for these inaccuracies were lag errors of the servomechanisms (despite a quite slow computation). Transformation in polar coordinates with resolvers has not proven itself, since α_K is subject to only minor fluctuations and at the same time the /9

*The computations on the analog computer were made through the kindness of Herr Dipl.-Math. H. Hentschel.

limited resolving power of the sine-cosine potentiometer becomes noticeable.

As a supplementary condition, the energy equation can be introduced; this is received by multiplying the vector equation of the translation scalar by the velocity vector $\vec{v}$ and multiplying the moment equation by ω and carrying out time integration for both. This energy equation introduced as a supplementary condition was used for improving the computations using the steepest descent method. Here also, as a result of differentiation for α_K, the value is approximated by a polynomial. If E is the energy, then we will have

$$E(\underline{x} + \delta\underline{x}) - E_0 = \varepsilon$$

Now $S = \varepsilon^2$ must be minimized. Then we will have

$$\frac{dS}{dt} = 2\varepsilon \frac{d\varepsilon}{dt} ,$$

and

$$\frac{d\varepsilon}{dt} = \frac{dE}{dt} = \sum_\alpha \frac{\partial E}{\partial x_\alpha} \frac{d\delta x_\alpha}{dt} ,$$

$$\frac{dS}{dt} = 2\varepsilon \sum_\alpha \frac{\partial E}{\partial x_\alpha} \frac{d\delta x_\alpha}{dt} , \qquad \frac{d\delta x_\alpha}{dt} = -A\varepsilon \frac{\partial E}{\partial x_\alpha} ,$$

with A being an amplification factor. For the system of differential equations, we then have:

$$\dot{x}_\alpha = X_\alpha(x') - A\varepsilon \operatorname{sgn} \frac{\partial E}{\partial x_\alpha} ,$$

that is, it is necessary to shift to the zero positions of the partial derivatives using a comparator. This is in some features the essence of the steepest descent method. These computations could generally not be carried out with the analog computer at our disposal because the outfitting with components did not suffice. The operation was simulated digitally using an Algol program.

5. Model Computations /10

As already mentioned, the computations were carried out for the special personnel glide surface parachute (Figure 2) and the following data were selected:

$R = 3{,}0\,m$, $s = 9{,}1\,m$, $G_L = 100\,kp$, $i_k^2 = 0{,}344\,m^2$.

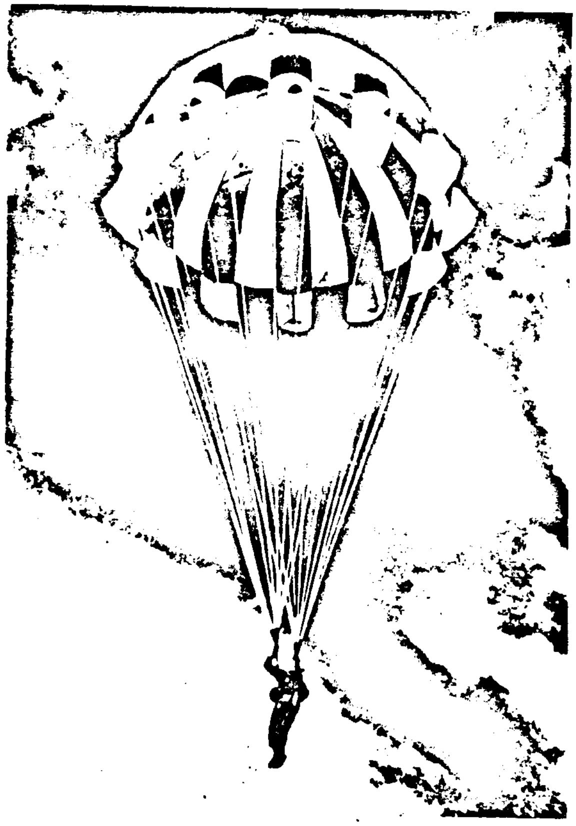


Figure 2. [Caption not Visible]

The moment of inertia (radius of inertia) can be determined if the shroud is mentally replaced by an ellipsoid of revolution of equal volume and this is enlarged [2]; the moments of inertia are formally known for an ellipsoid. Figure 3 shows the aerodynamic values determined from wind tunnel investigations of models. Figure 3 shows 3 cases of different porosity (the effective porosity is given as a dimensionless number, in accordance with the data given by H. G. Heinrich in [5] for geometrically uniform chutes. In particular, in the case of the impermeable chute ($\eta = 0$) we see that $\partial C_M/\partial\alpha$ in the neighborhood of $\alpha_K = 0$ is negative, that is, in this region the chute has no restoring moment.

As a typical result we will show a case (Figure 4) in which the chute is stable in the entire region of angles of attack. As expected, we obtain attenuated oscillations of a certain frequency. That the velocities V, V_K and V_x have a double frequency is easy to understand if the chute is regarded as a pendulum. The appearing minor amplitudes of oscillation of the shroud show, as also can be seen on the trajectory curves, that the load essentially oscillates about

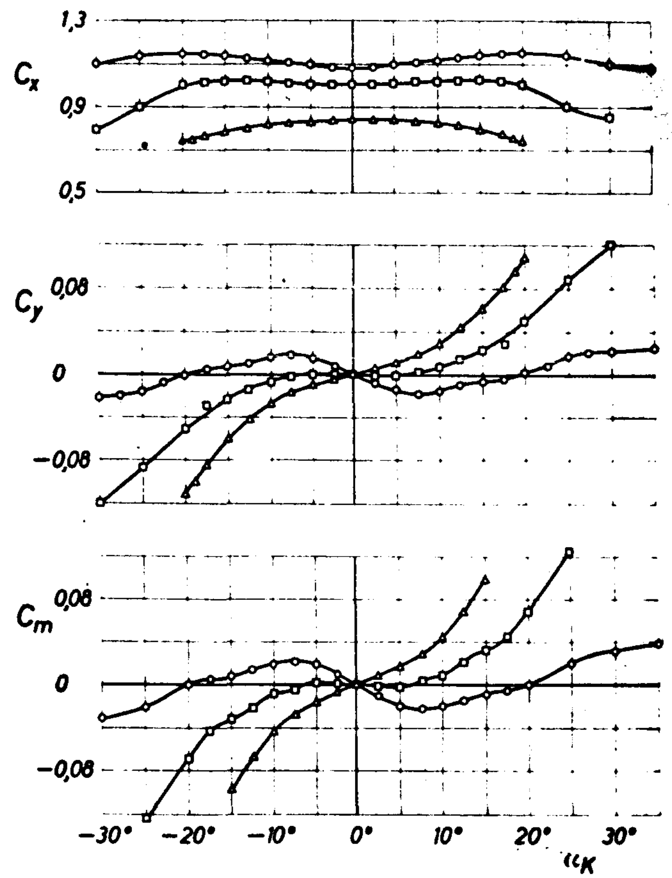


Figure 3. Aerodynamic Values for Personnel Glide Surface Parachute.

the shroud. These values, plotted as a function of time, only in the case of more exact study reveal deviations from the oscillation behavior of a linear system. This becomes clearer in a phase diagram (Figure 5) in which $\omega = \dot{\vartheta}$ is plotted as a function of ϑ. It can be seen clearly from the time marks plotted on the spiral that the duration of oscillation decreases with amplitude. It also is easy to learn from the amplitude ratios that attentuation decreases with amplitude.

Now we will consider the trajectory curves of the shroud and load (Figure 6), in which the chute is sketched in schematically at 1-second intervals; thus, we can vary the porosity (a,b,c), or with the same porosity we can vary the /11 length of the shroud lines (c,d,e). We find that with increasing porosity, the attenuation increases, with a lesser decrease of the duration of oscillation. The duration of oscillation increased, by analogy with a pendulum, with the length of the shroud lines. This is shown by the values V, V_y, ϑ and α as functions of time (Figure 7). Here, about 20 cases were investigated and the astonishing fact was discovered that the square of the duration of oscillation is proportional to the shroud-load distance. This suggests the possibility of

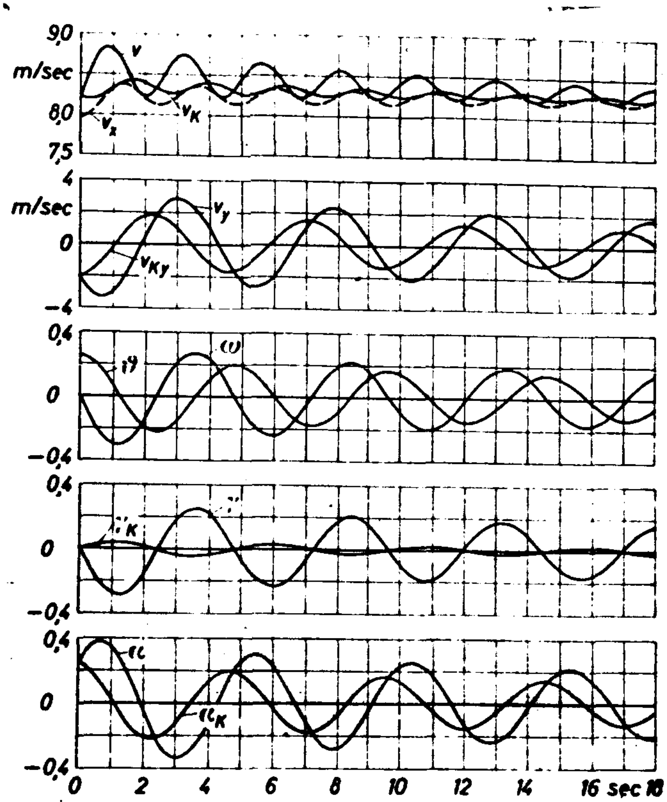


Figure 4. Example: Temporal Variation; η = 0.096, s = 9.1 m, ϑ_0 = 0.25.

representing the observed facts in an empirical formula, using the formula for a mathematical pendulum, supplemented by a proportionality factor. If we replace the resistance (drag) value C_{W_0} by the stable velocity of descent V_s, we obtain

$$T \approx C_{W_0} 2\pi \sqrt{\frac{s}{g}} = \frac{4\pi G_L}{\rho F V_s^2} \sqrt{\frac{s}{g}} \; .$$

The oscillation durations computed using this formula agree well with the model computations (Figure 7,b).

The attenuation is influenced to only a modest extent by change of the length of the shroud lines. In the case of a stable chute, there will be a weak maximum in the region of shroud lines of ordinary length.

The assumption concerning the entrained air mass requires further checking. The computations carried out (Figure 8) for different mass ratios μ_K with a constant load show that the duration of oscillation is virtually independent of

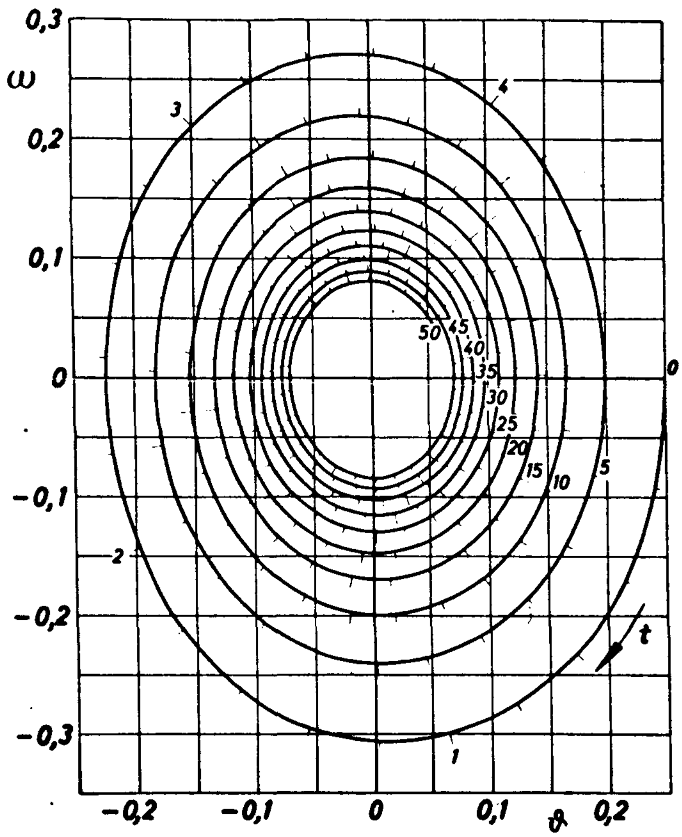


Figure 5. Example: Phase Diagram;
G_L = 100 kg, η = 0.096, ϑ_0 = 0.25.

the entrained air mass. The attenuation indeed is somewhat greater in the case of a smaller mass, but the initial deflection of the shroud also is initially enlarged.

In all of the cases considered until now, the air density ρ has been assumed constant, equal to that at the ground $\rho = \rho_0$. For the motion of a parachute at other air densities, that is, for other altitudes, the following assumptions can be made:

a) the aerodynamic values remain unchanged (that is, porosity does not change);

b) the entrained air mass should decrease in mass in such a way that the entrained air volume remains unchanged, that is

$$\mu_K = \frac{\rho}{\rho_0} \mu_{K0} \quad .$$

In computing a part of the trajectory, the air density again is held /12

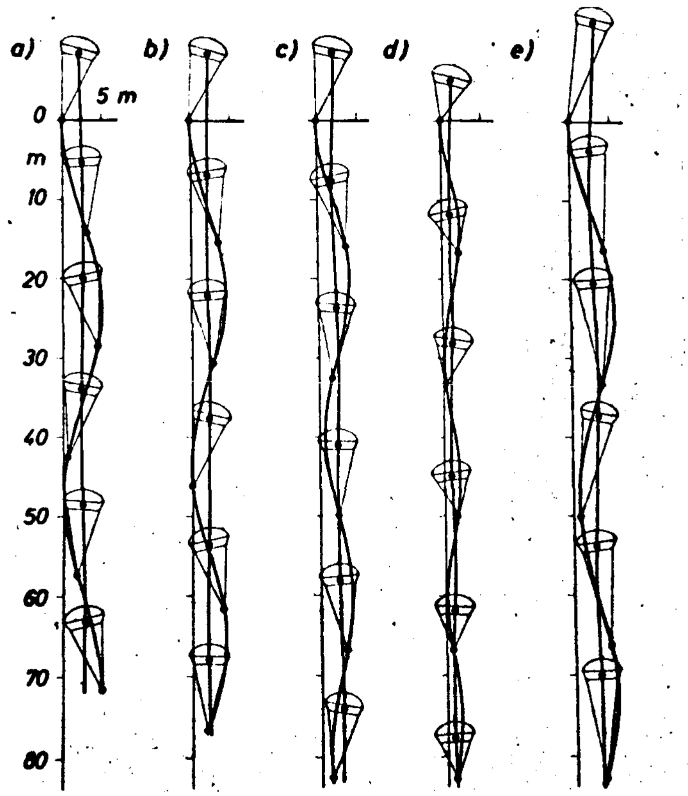


Figure 6. Example: Trajectory Curves. $\vartheta_0 = 0.25$; a) $s = 9.1$ m, $\eta = 0$; b) $s = 9.1$ m, $\eta = 0.042$; c) $s = 9.1$ m, $\eta = 0.096$; d) $s = 5.1$ m, $\eta = 0.096$; e) $s = 13.1$ m, $\eta = 0.096$.

constant. The computations show (Figure 9) that the lateral deflection of the shroud is greater than at the ground, the attenuation is somewhat greater and the duration of oscillation is less.

If we compare the trajectory curves for different cases in both stable and unstable cases (Figure 10), the initial conditions are decisive for the oscillation behavior. The stable chute has a vertical trajectory. In an unstable case, with a small initial deflection, the chute is deflected further. This results in a motion in which a lateral velocity component will be maintained, that is, the chute is driven sideways.

If, perchance, we study V_{Ky}, α_K and γ_K, we see (Figure 11) that, in general, an attenuated oscillation appears, but a stable condition of oscillation can be attained only if the chute has reached a position in which $\partial C_M/\partial \alpha_K$ is positive. In this case, the course of motion was followed over 200 seconds and, of course, the air density also was held constant here in order not to vary still another parameter. The position for which $\partial C_M/\partial \alpha_K = 0$ lies at $\alpha_K = 0.2$. Since the lateral velocity becomes constant, the motion becomes rectilinear at a certain angle to the vertical (about 20°).

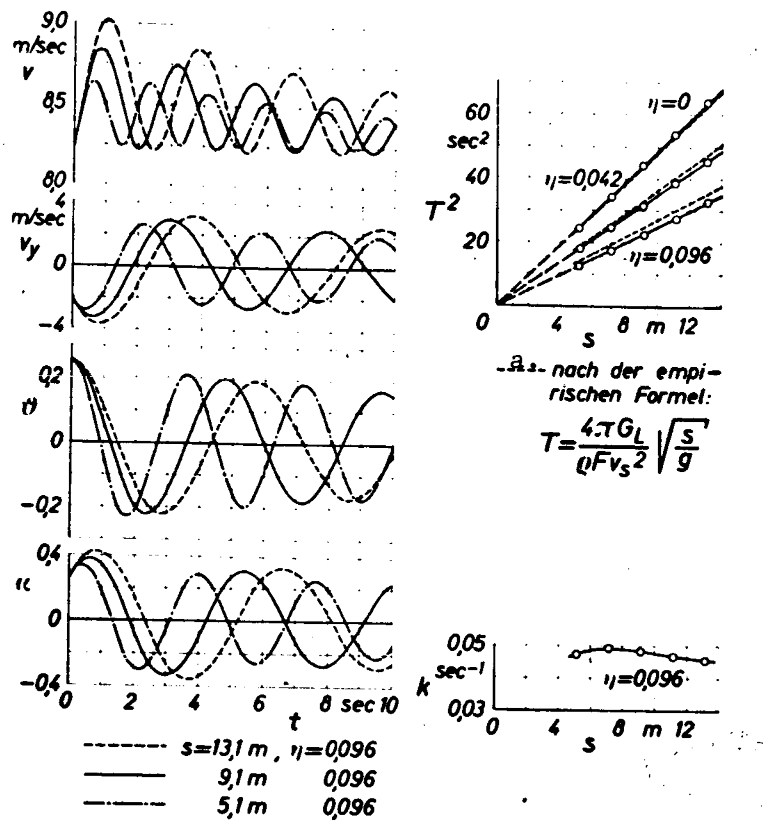


Figure 7. Example: Duration of Oscillation and Attenuation. Legend: a = On the Basis of the Empirical Formula.

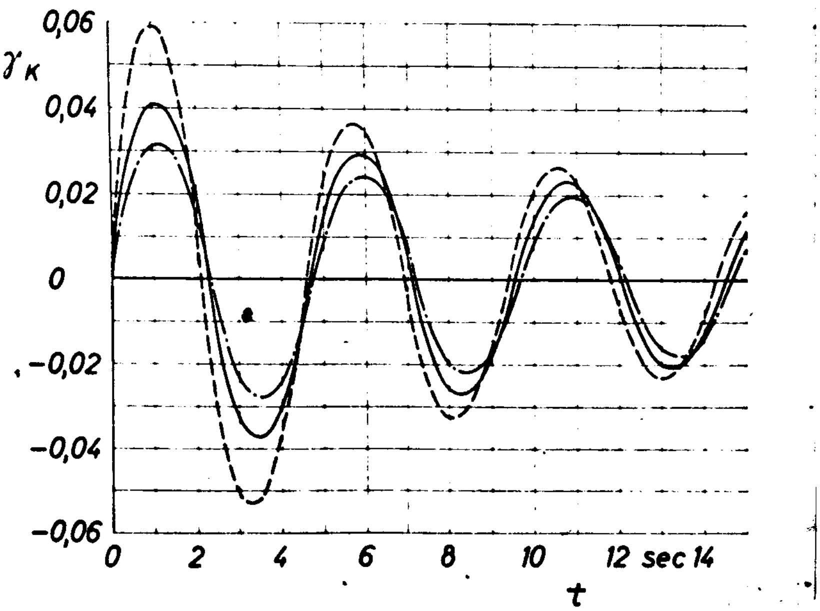


Figure 8. Example: Angle of Inclination of Trajectory for Different Entrained Air Masses; ---μ_K = 0.6; ___ μ_K = 1.0; _._ μ_K = 1.4.

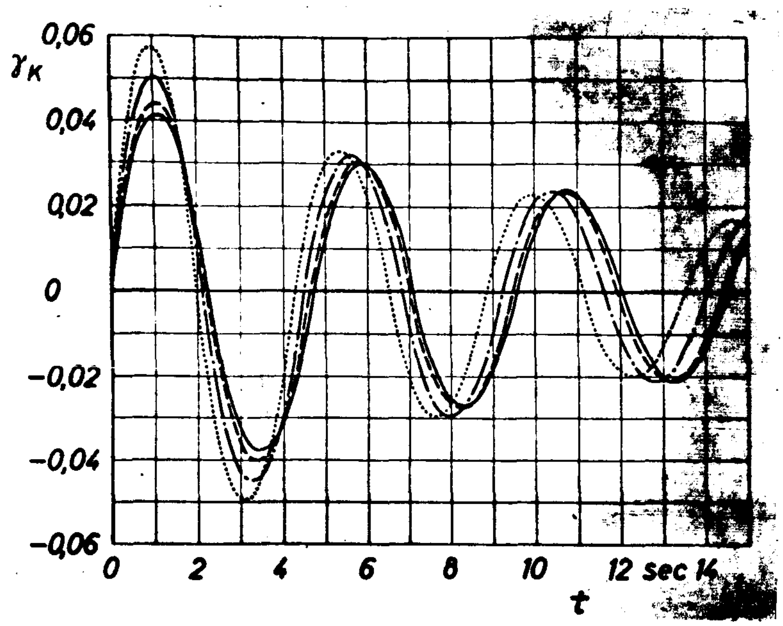


Figure 9. Example: Angle of Inclination of Trajectory for Different Altitudes. ___ H = 0 km, μ_K = 1.00; --- H = 2 km, μ_K = 0.822; - - H = 6 km, μ_K = 0.538; ... H = 10 km, μ_K = 0.338.

6. Summary

The computation of numerous examples of a special type (numerically about 80 cases were considered) shows that the oscillations of a parachute show a certain typical type of behavior which is characteristic of nonlinear oscillations. A qualitative agreement with experiments was achieved in a number of respects. Quantitative comparative investigations still could not be carried out because until now it still was not possible to carry out drop experiments with chutes of the considered type.

In addition to the information which the experimenter obtains on different properties of parachute oscillation, it appears to be particularly important that this is a case where for the investigation of the dynamic behavior a nonlinear computation is the only approach which can give an unobjectionable description of the process. The available experimental data, such as wind tunnel measurements for asymmetrical chutes in 6 components, measurements of the entrained air mass, etc., should be used in further broadening of theoretical investigations. /13

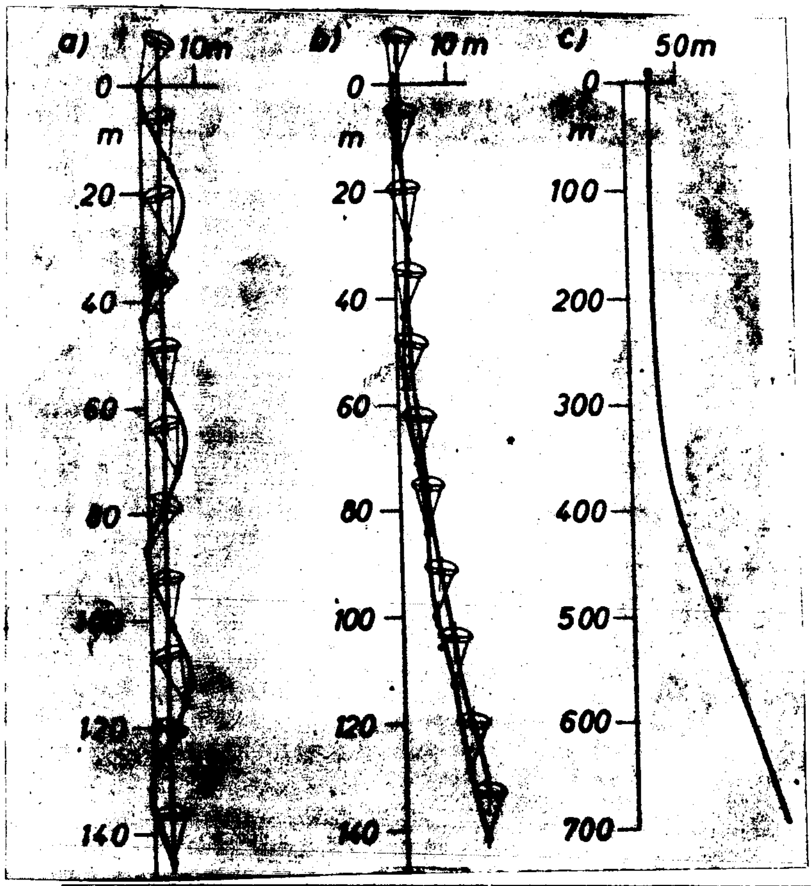
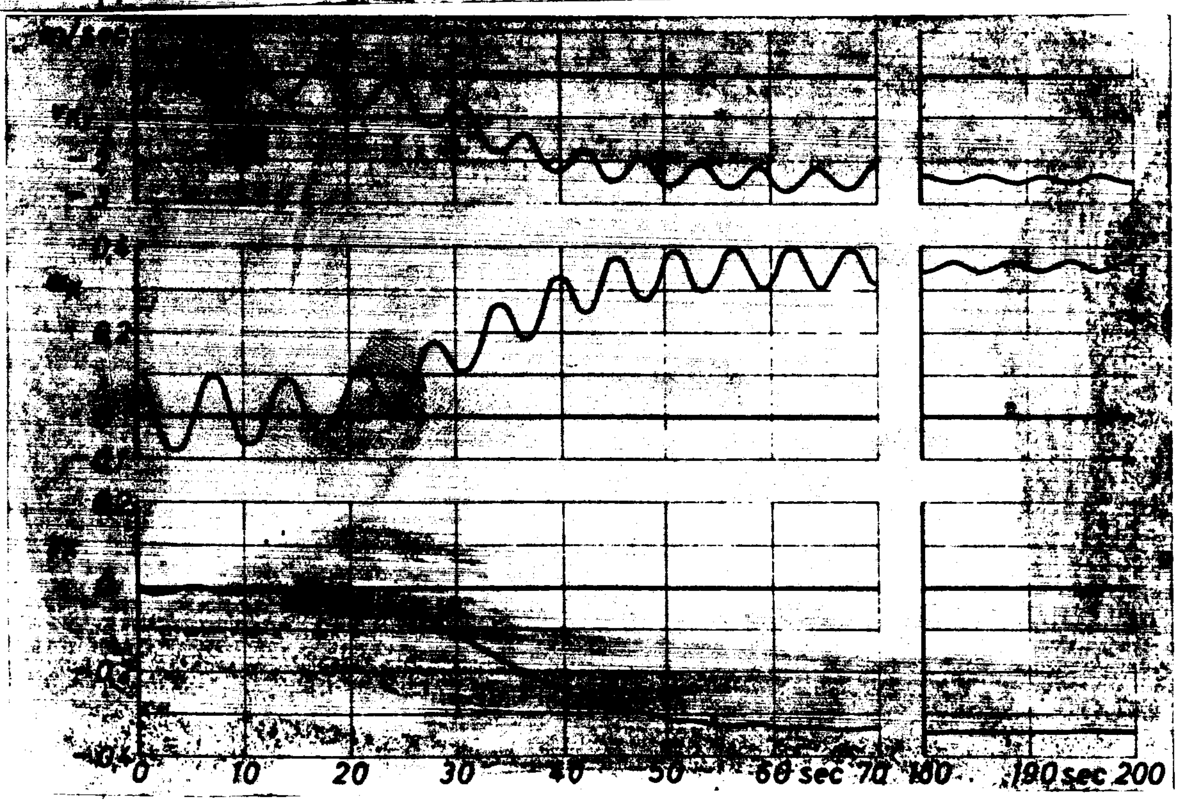


Figure 10. Example: Trajectory Curves. G_L = 1,000 kg; η = 0; a) ϑ_0 = 0.40; b) ϑ_0 = 0.10; c) ϑ_0 = 0.10.

Figure 11. Example: η = 0, ϑ_0 = 0.10.



REFERENCES

1. Brodetzky, S.: The stability of the parachute. The Tohoku Math. Journ. 14: 116-123, 1918.

2. Henn: Die Absinkeigenschaften von Fallschirmen. (The Drop Properties of Parachutes.) Zentrale für Wiss. Berichtwesen, Berlin-Adlershof, UM 6202, 1944.

3. Lester, W. G. S.: A Note of the Theory of Parachute Stability. Royal Aircraft Establishment TN No. Mech. Eng. 358, 1962.

4. Heinrich, H. G. and E. L. Haak: Stability and Drag of Parachutes with Varying Effective Porosity. Aeronautical Systems Division, TDR-62-100, 1962.

5. Ludwig, R. and W. Heins: Investigations on the Dynamic Stability of Personnel Guide Surface Parachutes. DFL Bericht No. 203, Braunschweig, 1963.

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