\[
\frac{c_2}{t} = 0.001 e^{0.001 t} x_2
\]
The electronic analogue computer is a powerful tool for the design and analysis of engineering and physical systems. A mathematical model set up on an analogue computer often enables basic design work to be completed with confidence before any prototype construction is started.

Its versatility is such that many applications exist in fields where analysis is often not complete or is even non-existent, and where "cut and try" methods have been used for too long.

This booklet contains a detailed account of the use of the new Solartron Analogue Tutor and a general description of analogue computer techniques and existing applications.

The Analogue Tutor Mk. II has been designed specifically for use by educational establishments to instruct the engineers and technicians of the future in the science of analogue computing which is becoming more widely used throughout all branches of industry.
General Introduction to Computer Techniques

Automatic Computation

Many recent advances in pure and applied research have been based at some stage on the application of electronic computers. Processes of analysis, design, development and operation of systems differing as widely as integrated guided weapon defence systems and the commercial operation of teashops have been subjected to the searching logic of computing techniques.

There are two essentially different approaches to automatic computation, which have been developed into two types of machine. The first type historically, represents physical variables continuously as equivalent physical variables of a different type. The second represents variables discontinuously as numerical approximations to the desired quantity, and operates on these numbers arithmetically. These types are known respectively as “analogue” and “digital” computers.

Computers of both types have been built without the use of electronic techniques but these are cumbersome, slow, and of limited application. For a very large proportion of engineering problems the high speed of electronic analogue computers permits dynamic analysis to be carried out at the actual speeds of the physical system under investigation, with normal accuracies which considerably exceed the precision of the available input data.

The electronic analogue computer represents all physical variables of any system under investigation as analogous variations in voltage, which are arranged to obey precisely the same relationships as the physical quantities. In general, the independent variable is time, so that the most suitable applications for electronic analogue computers are those which involve the analysis of the behaviour of dynamic systems. Both steady state and time dependant solutions are readily obtainable.

Further, the analogue computer can provide a direct electrical equivalent of physical systems, which provides a deep insight into the dynamic behaviour of physical systems.

The analogue computer may be used both as a computer and as a simulator, these terms often being used more or less interchangeably. Strictly, when used for system analysis the term “computer” applies, and when used to represent some known part of a physical system to permit analysis of the behaviour of some other portion, the term “simulator” is more correct.
**General Description of the Tutor**

The Solartron Analogue Tutor consists of two bench cabinets. One is the power supply AS.1403. The other contains:

- 6 Operational (3 Dual) Amplifiers type AA.1054. Four of these operate as either Summers or Integrators, selected on associated switches. Each Summer/Integrator has—
  - 2 input resistors, $1 \times 1\, \text{M ohm} \times 100\, \text{K ohm} \times 0.5\%$
  - 1 feedback resistor, $1\, \text{M ohm} \times 0.5\%$
  - 2 feedback capacitors, $1 \times 0.1\, \mu\text{F} \times 1 \times 0.01\, \mu\text{F} \times 1\%$

The other two amplifiers are available as summers only and each has:

- 3 Input Resistors, $2 \times 1\, \text{M ohm} \times 1 \times 100\, \text{K ohm} \times 0.5\%$
- 1 Feedback Resistor, $1\, \text{M ohm} \times 0.5\%$

6 Free resistors are also incorporated, $2 \times 1\, \text{M ohm}$ and $4 \times 100\, \text{K ohm} \times 0.5\%$

8 10-turn helical potentiometers of better than 0.01% resolution, $30\, \text{K ohm}$ each, two earth-free.

A 10-turn master reference potentiometer of 0.1% linearity, with calibrated dial.

A centre zero nulling meter with three sensitivity ranges $\pm 100\, \text{V}, \pm 10\, \text{V}$ and $\pm 1\, \text{V}$.

A computer operation switch with operational control modes: Pot Set, Problem Check, Compute, Hold, Repetitive Slow, Repetitive Fast and Slave.

A patch panel system with removable problem board. The patch panel connections are defined on colour coded cards printed with circuit schematics. These are available in quantity at low cost, so that a card for each problem may be stored for future use or for easy cross reference during instruction.

An external meter jack to facilitate use and demonstration of a Digital Voltmeter for setting up purposes.

An internal reference supply $\pm 100\, \text{V} \times 50\, \text{mA} \times 0.25\, \%$

Two diode pairs for simulation of simple discontinuities.

Each power supply is capable of driving up to four Tutors thus reducing expansion costs. As an alternative, a small assembly of Multipliers and/or Function Generators (three in total) may be substituted in place of additional Tutors. Connection of these units into the computer programme would be via the 20 Trunk lines on the patch panel. Four Tutors may be operated simultaneously on one larger problem, thus providing for a maximum computing capacity of 24 amplifiers (16 Summer/Integrators and 8 Summers).
Description of Patch Panel Layout

The patch panel is laid out diagrammatically to indicate clearly the function of each patching point. These points are also coloured and the coding is as follows:

**Blue**—Amplifier inputs (including initial condition inputs to integrators). The gain on each input is marked either 1 or 10, and the resistor value is also marked in megohms.

**Yellow**—Amplifier outputs. (five for the Summer/Integrators and three for the Summers).

**Orange**—Potentiometers. The high end (input) of the potentiometer is marked H and the arm marked A (output). Two potentiometers have all three terminals appearing on the patch panel and are marked H, A and L.

**Green**—Amplifier summing junctions. In the case of the two summing amplifiers A2 and B2 the top green hole is the true summing junction and the bottom hole is the common terminal of the input resistors. The integrators have three green terminals. The choice of feedback capacitor is determined by the appropriate linking of these terminals.

**Red**—+100V reference supply, except for the two adjacent red holes at the bottom centre of the panel. These must be linked in order to connect the H.T. supplies. Removal of the patch panel will disconnect the H.T. supplies automatically.

**Blue**—Negative 100 volts reference supply (labelled −).

**Black**—Signal ground.

**White**—Block of 20 holes are the trunk connections to external equipment.

**White**—Strips joined by black lines are busbars i.e. multiples.

**White**—Vm—input to voltmeter monitoring circuit.

**White**—HOLD—when the negative 100 volts is applied the computer is switched to the HOLD condition.

**Note**

The free components have blue/yellow terminations and the resistor values are marked 1 MΩ or 0.1 MΩ.

The diode pairs have red-green-blue terminations, where the green hole is in the junction of the pair.
Applications of Analogue Computers

As stated before, analogue computers are particularly useful for analysis of the dynamic behaviour of physical systems. Any mechanical, electrical, biological, or even economic system which involves motion or variation in time may be studied. Provided that the equations defining the behaviour of the various parts of the system may be formulated, then an exactly analogous system may be set up on the computer, and the overall behaviour of the system studied even before the system in whole or in part, exists physically.

For example, a proposed bridge design may be studied in respect of its behaviour under wind loads and modified if necessary to ensure its aerodynamic stability under any conceivable conditions, thus avoiding the type of disaster which overtook the Tacoma Narrows Suspension Bridge in the U.S.A. soon after it was completed.

With the rapid advances in engineering applications of automatic systems which, in general, involve the operation of the principles of feedback, all stages from the initial conception to the finished system may be analysed to ensure that the system performance will meet the design requirements. Thus all branches of control and regulator engineering have immediate applications. All production processes in which operation is based on continuous flow methods may be studied with the aim of improving the operating conditions; this includes not only flow chemical production and refining processes, but also generation of electrical power and similar systems.

Dynamic mechanical systems are readily amenable to this type of analysis, for example structures under vibration conditions, car suspension systems, aircraft in flight (and small sections of aircraft), hydro-electric power generation, water flow conditions, and so on.

On a smaller mass scale, the stability analysis of the beams in particle accelerators may be studied, with the aim of improving the dynamic focusing system.

A wide range of applications is opened up by using analogue computer networks to simulate components at the design stage in systems incorporating many units, some of which exist. For example, it is possible to simulate the control unit of a motor system using the actual motor-alternator unit with an artificial load in order to establish the required performance of the control unit, before the latter is built.

A further class of applications may be described as "Data Reduction" processes. For example, in a large chemical plant signals may be available representing pressures, temperatures, mass flows, and so on at various points. The production rate will normally bear some mathematical relationship to these quantities and an analogue computer may be used to accept these signals as inputs, solve the relevant equations and produce outputs, for production monitoring or control purposes, representing, for example, production rates or efficiencies. In these applications the output information is simply a digested and converted version of the input. In many cases in this type of application the computer in fact represents part of the process control system, when some output signals are arranged, through suitable electromechanical transducers, to alter the operating behaviour of the main process, e.g. by altering valve positions, temperature controller set points, or material input feed rates.

Analysis of the behaviour of systems involving distributed parameters, for example, multidimensional heat flow systems, in which partial derivatives with respect to spatial co-ordinates rather than time occur, can only be dealt with in special cases, using difference equation methods. Even in these cases, however, the analogue computer techniques eliminate much tedious calculation for results of comparable accuracy.

An exhaustive list of possible applications is not practical, in view of the wide scope of this type of analysis, but a short series of types of use is given below:
1. Optimisation of parameter values in system design studies.
2. Dynamic behaviour of open and closed loop control systems.
3. Simulation of components in otherwise existing systems.
4. Stability of structures under dynamic load conditions.
5. Data reduction.
Analogue Computer Techniques

As described before, the applications of analogue computing techniques cover a wide range of research and development problems. Reduction of any one problem to a form in which it may be programmed for, and analysed by, an analogue computer almost invariably requires initially a statement of the system behaviour in analytical terms. That is, all problems have to be reduced to a system of equations which may then be interpreted in computer terms.

However, when complex systems have to be simulated, overall system equations are not required, since the individual components of the system may be set up as such, and interconnected analogously in the computer to their physical interconnections.

Clearly, the more elaborate the system under investigation, the greater the computer capacity required to simulate it. Thus the complete simulation of a guided missile in flight will require a large and complex computer, but the behaviour of a car wheel under road surface variations requires only a small and simple computing arrangement.

All analogue computers consist of an assembly of identical units of a few different basic types which are used for the fundamental mathematical operations of algebraic summation, multiplication and division by constants, multiplication and division of variables, and integration with respect to time. Other units are usually available for the more elaborate operations, generations of powers, integral and otherwise, trigonometric functions, logarithmic and exponential functions, empirical functions of one, or sometimes two, variables, and so on.

Interconnection of these units will permit the generation of networks solving any linear or non-linear differential equations of any order and degree, provided that all differentiations and integrations are with respect to time. Worked examples at the end of this brochure show how these mathematical operations are performed accurately using feedback methods in conjunction with electronic amplifiers.

Provision is made in complete computers for all initial boundary conditions to be applied. The system under investigation may then be analysed by the application of any suitable input forcing functions of time, for example, unit pulse, step, or square-wave functions for transient response analysis; sinusoidal functions of various frequencies, for frequency response analysis; or random noise signals for investigation of noise responses.

Courtesy of U.S.I.S.
Switching on Procedure

Before switching on ensure that the following patch-panel connections are made:

1. The HT link.
2. The feedback resistor to the output for each of the two summing amplifiers.
3. The appropriate resistor or capacitor to the summing junction of each summer/integrator.

Connect the Tutor to the AS1104.2 power supply using one of the four outlets provided (up to 4 Tutors or non-linear racks can be coupled to one AS1104.2). Then connect the AS1104.2 to the mains supply. Switch on the heater supplies using the switch located on the front of the power supply. Allow a few minutes for warming up and then switch on the HT.

Individual Tutors can be switched off simply by removing the patch panel.
The Basis of Analogue Computing

(a) Linear Operations

Summation

The basis of accurate mathematical operations with voltage signals as variables is the use of negative feedback applied to electronic high gain voltage amplifiers.

Consider the circuit shown in Diagram 1.

The triangle represents an electronic amplifier with the input terminal at the base and the output terminal at the apex. Its gain extends to zero frequency and it is arranged internally to have its output terminal at zero volts with respect to earth when its input terminal is at earth potential. (The zero-setting potentiometer is situated on the front panel of the instrument).

The voltage gain is designated $-A$, the negative sign indicating that a positive input signal gives a negative output signal. All voltages are measured with respect to earth.

Then we have:\[ V_o = -A \cdot e \]

and by Kirchhoff's laws:

\[ \frac{V_i - e}{R_1} + \frac{V_o - e}{R_o} = 0. \]

Substituting for $e$ in the second equation:

\[ \frac{V_i}{R_1} + \frac{V_o}{AR_1} + \frac{V_o}{R_o} + \frac{V_o}{AR_o} = 0. \]

Hence

\[ V_o \left( \frac{1}{AR_1} + \frac{1}{AR_o} + \frac{1}{R_o} \right) = -\frac{V_i}{R_1}. \]

Now if $A$ is made indefinitely large:

\[ \frac{V_o}{V_i} = -\frac{R_o}{R_1} \]

Thus the output voltage can be made equal to the negative of the input signal multiplied by any arbitrary constant $R_o$.

The number of input resistors connected to the amplifier can be greater than one, see Diagram 2.

By the above methods of analysis the following equation results:

\[ V_o = -R_o \left( \frac{V_i}{R_1} + \frac{V_o}{R_2} + \frac{V_o}{R_3} \right) \]

and the basis of summation is established.
**Multiplication by a constant**

This function is provided by manually adjusted potentiometers arranged as shown in Diagram 3.

Clearly, as the potentiometer is adjusted from $\theta = 0$ to $\theta = 1$ the proportion of $V_1$ appearing at the wiper will vary from zero to unity. Hence readily adjustable factors less than unity can be inserted. The equation is:

$$V_o = -\frac{R_o}{R_1} \cdot \theta$$

$$V_1 = -\frac{R_o}{R_1} \cdot \theta$$

If the potentiometer is connected between the output of the amplifier and the output end of $R_o$ as shown in Diagram 4, then:

$$V_o = \frac{R_o}{R_1} \cdot 1$$

$$V_1 = -\frac{R_o}{R_1} \cdot \theta$$

Thus division by a constant is established.

**Note**

In practice the multiplication factor $\theta$ is modified by the loading of the potentiometer wiper. The procedure for setting a potentiometer automatically adjusts for wiper loading. Care should be taken when setting a potentiometer to ensure that the wiper is loaded as in the problem under investigation.

**Experiment 1**

Summation and multiplication by a constant.

Using one of the summing amplifiers (either A2 or B2) patch the circuit where PA2 is the potentiometer associated with amplifier A2, etc.

Set the potentiometers to the following values using the reference potentiometer and null meter.

- $PA2 = 0.5$
- $PA3 = 0.3$
- $PB2 = 0.15$

Then $V_o = -(0.5 + 0.3 - 1.5) = +0.7$ i.e. +70 volts

Check that the output of A2 reads +70 volts within 2% (maximum error using four resistors—The error in setting the potentiometers is an order less).

The procedure for potentiometer setting is as follows:

1. Turn Reference potentiometer to the selected value of coefficient.
2. Switch Reference Potentiometer selector to $+$.
3. Operate the potentiometer setting switch for the appropriate potentiometer. One such switch is mounted to the left of each pair of potentiometers, switch this up for the upper potentiometer and down for the lower potentiometer. Operation of this switch connects +100 volts to the high end of the potentiometer and connects the arm to the meter circuit.
4. Turn the potentiometer until the meter reads zero, increasing the sensitivity as necessary using the switch situated below the meter. The most sensitive position is "NULL".
Integration

An important result occurs when the feedback impedance of an amplifier is purely a capacitor, as shown in Diagram 6.

The transfer function is then:

\[
\frac{V_o}{V_i} = -\frac{1}{pCR}
\]

where \( p \) is the differential operator \( \frac{d}{dt} \)

or alternatively,

\[
V_o = -\frac{1}{RC} \int V_i \, dt
\]

Hence the mathematical process of integration with respect to time is established. Provision is made for an initial voltage to be applied to the output of the amplifier, when used as an integrator, so that the arbitrary constant of integration (i.e. \( V_o \) at \( t = 0 \)) can be inserted when required.

Experiment 2

Integration (for this experiment some type of recording equipment is necessary such as an oscilloscope or X-Y recorder).

Switch A1 to "Integrate", (SUM/INT switch to INT) connect up the circuit shown in Diagram 7.

The 0.1\( \mu \)F capacitor is selected on the patch panel by linking the appropriate green holes.

Set PA1 to a coefficient of 0.1 by the method described in Experiment 1. The integrator when in compute will take one second to reach 100 volts. This is a little fast for manual operation and the computer should be switched to Repetitive. There are two repetitive positions on the MODE switch, RS (repetitive slow) and RF (repetitive fast). There is also a concentric potentiometer enabling the repetitive speeds to be continuously varied in the ranges 0.8 seconds to 2.5 seconds (RS), and 0.08 seconds to 0.25 seconds (RF).

Connect output of A1 to the X plates of a D.C. oscilloscope via potentiometer A2. Zero the oscilloscope spot. Operate the key switch associated with A2 and adjust PA2 until the spot moves to full scale. Hence + 100 volts at the output of amplifier A1 moves the spot to full scale. Return A2 potentiometer switch to the central position.

Switch the computer to the slow repetitive state and adjust the repetitive speed potentiometer until the spot moves the full width of the oscilloscope.

An accurate time base, synchronised with the computer has thus been established.
Experiment 4

A mechanical system consisting of a mass connected to a spring and dashpot is described by the equation:

\[
m \frac{d^2x}{dt^2} + k \frac{dx}{dt} + sx = + mg.
\]

where \( x \) = displacement of mass
\( m \) = mass
\( k \) = damping constant
\( s \) = stiffness force/unit extension.

The method of setting up analogues for all equations of this type is as follows:

1. Segregate the highest order derivative term on the LHS of the equation:
\[
m \frac{d^2x}{dt^2} = -k \frac{dx}{dt} - sx + mg
\]
2. Add all the terms on the RHS using a summing amplifier.
3. Integrate the output of this summing amplifier the necessary number of times.
4. Feed back the various derivative terms through the required coefficients with due regard to signs.

The arrangement of the summing unit is then shown in Diagram 11.

\[
V_o = \frac{k}{m} \frac{dx}{dt} + sx - mg = -\frac{d^2x}{dt^2} \quad \text{(from equation)}
\]

\( V_o \) is then integrated twice, bearing in mind that each amplifier produces a sign inversion.

The outputs of the integrators are \( \frac{dx}{dt} \) and \(-x\) respectively.

The complete circuit is then shown in Diagram 13. The scaling is calculated by the method used in Experiment 3, the starting point for the scaling is with the full scale value of \( x \) at 1 metre.

The known constants are:
\( m = 1 \) Kg
\( g = 9.81 \) m/\( \text{s}^2 \)

\( k \) and \( s \) are parameters under investigation and are assigned full scale values of 100 Kg/m/sec and 10 Kg/m respectively.

The time-base for the oscilloscope or X-Y plotter should be connected as in Experiment 2. Obtain plots of \( \frac{dx}{dt} \) and \( \frac{d^2x}{dt^2} \) for various values of \( s \) and \( k \), and also for different initial values of \( x \). Note that if \( x_0 \) is zero then the mass still falls and oscillates due to the gravity term.
$V_0 = -\frac{d^2 x}{dt^2}$
(b) Non Linear Operation

**Experiment 5**

Limiter circuit—using one pair of the built-in diodes. The circuit arrangement is shown in Diagram 14.

The potentiometers P and Q are the "Free" potentiometers situated in the right half of the patch panel. The positive limit is adjusted by potentiometer P and the negative limit by Q. The input/output response of the circuit is given in Diagram 15, for $-100 \leq V_i \leq +100$.

To obtain $V_i$, sweeping from $-100V$ to $+100V$ use the circuit of Experiment 2 but make the initial condition at the output of the integrator $-100V$. Reset potentiometer PA1 to 0.2.

**Experiment 6**

Many types of engineering problems involve non-linear elements giving rise to discontinuities in transfer gains, for example, at acceleration, velocity, or position overload points in mechanical displacements.

Such non-linearities may be treated using methods based on diodes as components in computing impedances. As an example, consider the network shown in Diagram 16. In this case, for values of $V_1$ such that $V_2$ is numerically less than $Vb-$ or $Vb+$, both diodes are non-conducting, when:

$$V_a = -V_1 \frac{R_1}{R_1 + R_2}$$

If $V_2$ reaches either $V_{b+}$ or $V_{b-}$ the corresponding diode conducts, and, for $D_1$, the input signal is then attenuated by $R_3$ in parallel with $R_1$.

It can readily be shewn that, with $D_1$ conducting

$$\frac{V_a}{V_i} = \frac{R_2 \cdot R_0}{R_1 \cdot R_2 + R_2 \cdot R_3 + R_3 \cdot R_1}$$

or with $D_2$ conducting

$$\frac{V_a}{V_i} = \frac{R_1 \cdot R_0}{R_1 \cdot R_2 + R_2 \cdot R_3 + R_3 \cdot R_1}$$

These expressions may appear cumbersome but, usually, the resistors may be chosen in reasonably simple ratios.

The overall behaviour of this arrangement may be shown thus:

- **Slope 1**: $V_a = -R_4 \cdot R_0/(R_1 \cdot R_2 + R_2 \cdot R_3 + R_3 \cdot R_1)$
- **Slope 2**: $V_a = R_0/(R_1 + R_3)$
- **Slope 3**: $V_a = -R_0/(R_1 \cdot R_2 + R_2 \cdot R_3 + R_3 \cdot R_1)$

Combinations of networks of this type enable straight line approximations to any desired, single valued, functions to be generated.

For these purposes the Solartron Analogue Tutor incorporates two pairs of diodes, and six isolated resistors.
Experiment 7—Generation of the Fresnel integrals

These integrals are of the form:

\[ F_1 = \int_0^{x_1} \sin (x^2) \, dx \quad \text{and} \quad F_2 = \int_0^{x_1} \cos (x^2) \, dx \]

As time is the only independent variable possible in analogue computation, \( t \) is substituted for \( x \) in the above expressions. (However see references at the end of this book, for the solution of partial differential equations using special techniques.) Diagram 19 shows the circuit diagram for the generation of these expressions and a Cornu Spiral is obtained by plotting \( F_1 \) against \( F_2 \) on an oscilloscope. The computing time is limited to \( t = 2.5 \) seconds (maximum repetitive time of the Tutor) so only a part of the spiral is obtained but there is sufficient for a good display.

In order to obtain the spiral in the other quadrant reverse the sign of reference voltage connected to potentiometer APA1 and take the input to potentiometer APB1 from an additional inverting amplifier connected to the \( \cos kt^2 \) output (amplifier BB1).

Notes on nomenclature used:
- AA1—Tutor A amplifier A1
- APA1—Tutor A, potentiometer associated with A1
- BT12—Tutor B, trunk connection 12.

The trunk connections used on each of the computers are shown in Diagram 20. The connections to both multipliers appear on the trunks in Tutor B, these are:

<table>
<thead>
<tr>
<th>Trunk</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( x ), input</td>
</tr>
<tr>
<td>2</td>
<td>( y ), input</td>
</tr>
<tr>
<td>6</td>
<td>(-y), input</td>
</tr>
<tr>
<td>7</td>
<td>product ( x \cdot y ),</td>
</tr>
<tr>
<td>3</td>
<td>( x_1 ), input</td>
</tr>
<tr>
<td>4</td>
<td>( y_1 ), input</td>
</tr>
<tr>
<td>8</td>
<td>(-y_1 ), input</td>
</tr>
<tr>
<td>9</td>
<td>product ( x_1 \cdot y_1 ),</td>
</tr>
<tr>
<td>5</td>
<td>(+100) to non-lin. units</td>
</tr>
<tr>
<td>10</td>
<td>(-100) to non-lin. units</td>
</tr>
</tbody>
</table>

**THE TWO QUARTER SQUARE MULTIPLIERS**

**TEN TRUNKS CONNECTED 1 TO 1**

X - INDICATE
THE TRUNKS USED
FOR CONNECTIONS
BETWEEN TUTOR A & B
(c) Advanced Problems using Two Tutors and a Non-Linear Extension

The following experiments can be performed using an extended Tutor consisting of:

- 2 Tutor units TY1351
- 1 Non-linear extension
- 1 Power supply AS1403

The computing capacity available is then:

- 8 Summer/Integrators (2 inputs each)
- 4 Summers (3 inputs each)
- 3 non-linear units (multipliers, function generators, diode resolvers or any combination)
- 4 Diode pairs
- 12 Free resistors
- 16 Potentiometers (4 earth free).

The connectors at the rear of the Tutor cabinet are as follows:

- Power supply plug.
- Control lines and Trunks 1-10.
- Duplicated control lines and Trunks 11-20.
- Coaxial socket for oscilloscope triggering.
- Jack socket for an external voltmeter.

The arrangement for connecting two Tutors and one non-linear unit is shown in Diagram 18. Each non-linear unit requires 12 trunk connections and in Diagram 18 these have been split taking four to Tutor A and eight to Tutor B. This means that if the non-linear unit contained, for example, three Quarter Squares Multipliers (TR1361), two would be available on the patch panel board in Tutor A and one in Tutor B. Similarly for the alternative non-linear modules such as function generators (TR1221.2) or diode resolvers (TR1360).
COMPUTER CIRCUIT DIAGRAM
Experiment 8. Rocket Landing Simulation

This topical experiment concerns the programming of the Tutor to solve the equations of motion of a rocket vehicle descending vertically on to the surface of the Moon, controlled by reverse thrust of the rocket motor. A throttle control (potentiometer) is provided for manual operation by the "astronaut".

The equations to be solved are given below and the computer flow diagram shown in Diagram 21.

Descent Conditions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total vehicle weight, less landing fuel</td>
<td>10 Kg</td>
</tr>
<tr>
<td>Landing fuel weight (parameter)</td>
<td>6 × 10⁻¹ Kg</td>
</tr>
<tr>
<td>Effective motor exhaust velocity</td>
<td>10Km/sec.</td>
</tr>
<tr>
<td>Maximum motor mass flow</td>
<td>300Kg/sec.</td>
</tr>
</tbody>
</table>

Hence peak vehicle thrust acceleration at "all burnt" 30 m/sec.²

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial height</td>
<td>100 Km.</td>
</tr>
<tr>
<td>Initial velocity downwards (parameter)</td>
<td>1 Km/sec.</td>
</tr>
</tbody>
</table>

The equations of motion to be simulated are derived as follows, all heights, velocities and accelerations being taken as positive upward.

By Newton’s laws:

\[ \text{Force} = \text{Mass} \times \text{acceleration} = \text{Rate of change of momentum.} \]

Hence

\[ (M_s + M_F) \frac{d^2h}{dt^2} = v \frac{d}{dt} (M_s + M_F) - g_m (M_s + M_F) \]

and

\[ (M_s + M_F) = M_s + (M_F)_i - \int_0^t m \, dt \]

therefore

\[ (M_s + M_F) \frac{d^2h}{dt^2} = (-r) \times (-m) - g_m (M_s + M_F) \]

where:

- \( M_s \) = Landing Mass of vehicle
- \( M_F \) = Instantaneous Fuel Mass (Landing Tank)
- \( (M_F)_i \) = Initial Fuel Mass (Landing Tank)
- \( h \) = Height above Moon surface
- \( v \) = Rocket motor exhaust velocity
- \( g_m \) = Gravitational acceleration at surface of moon (assumed constant)
- \( m \) = Fuel mass flow per second (set by throttle control)

To shorten the time taken for each landing the time scale should be speeded up by a factor of 10.

The drawing of the actual computer circuit diagram, the scaling, and the design of the detector circuits has been left as an exercise for the student. The zero height detector should be made to operate the computer "HOLD". The landing velocity can then be noted at the output of the appropriate amplifier. The object of the simulation is to determine, experimentally, the manner in which the "throttle" control has to be manipulated to give a smooth landing, with maximum fuel economy. The 6 × 10⁻¹ Kg of fuel allowed is more than is necessary to effect a perfect landing.

Notes
1. Division is achieved by coupling a quarter-square multiplier in the feedback of an amplifier.
2. There is an error in the circuit diagram around the front cover, have you noticed it?
Recommended Text Books

THE SOLARTRON ELECTRONIC GROUP LIMITED
Victoria Road, Farnborough, Hampshire, England
Telephone: Farnborough (Hants) 3000.
Telex: 8545 Solartron Fnb."
\[
\frac{dx_1}{dt} = 0.1 x_2, \quad \frac{d\eta}{dt} = x_1
\]